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Evaluation of the Stability Derivatives Using the Sensitivity Equations

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In this paper, we present a numerical method to predict the static and dynamic stability derivatives of the aircraft by solving the flow governing equations and the associated sensitivity equations. The aerodynamic model and the derivation of the sensitivity equations are discussed in detail. The key features of the present method are that the unsteady effects are taken into consideration and the complete set of stability derivatives, both static and dynamic, can be obtained simultaneously. The flow and sensitivity equations are solved using a computational fluid dynamics technique. The stability derivatives of a NACA 0012 airfoil undergoing a pitching oscillation are computed based on the solutions of the sensitivity equations. The present method is verified by comparing the aerodynamic forces obtained from the unsteady flow simulation with those predicted by the stability derivatives.

I. Introduction

The design of atmospheric vehicles and their control systems is a challenging area of ongoing research. The quality of a design is evaluated by the behavior of the aircraft when performing required tasks and maneuvers. For this purpose, one must be able to calculate the aerodynamic forces (and moments) acting on the aircraft at any instant of the flight. Because the aerodynamic forces depend on the aircraft motion history and there are infinitely many aircraft maneuvers, it is not viable to analyze the behavior of an aircraft unless a mathematical model for the representation of the aerodynamic forces is introduced. Up to now, the most widely used mathematical model is to express the aerodynamic forces as functions of the stability derivatives, which can be traced back to the work of Bryan [1] at the beginning of the 20th century.

Even though the mathematical models using the concept of the stability derivatives are usually simple, it is far from trivial to determine these stability derivatives because of the complexity of the flowfields and the lack of technique for "separating" the stability derivatives from the aerodynamic forces or from the combinations of the stability derivatives. Traditionally, the stability derivatives are evaluated through wind-tunnel experiments, flight tests, or empirical methods. The wind-tunnel experiments and flight tests are very expensive and time-consuming. The empirical methods are limited to certain stability derivatives and are sometimes not reliable. In recent years, there have been persistent efforts toward developing the computational fluid dynamics (CFD) technique for evaluating the stability derivatives numerically [2–4]. These methods are usually based on the solution of the sensitivity equations [5] that have been widely used in many fields such as aerodynamic optimizations.

In this paper, a numerical method for evaluating the stability derivatives using the equations of sensitivity is presented. This method is similar in spirit to that of Limache and Cliff [3] and Limache [4]. However, the present method allows the computation of dynamic stability derivatives by solving the unsteady flow governing equations and the unsteady sensitivity equations, whereas the method of Limache and Cliff [3] and Limache [4] only computes the static stability derivatives by considering the aerodynamically steady flows. After presenting the aerodynamic model and sensitivity equations in the next section of the present paper, we will apply this method to compute the stability derivatives of a NACA 0012 airfoil undergoing the pitching oscillations. The present method is verified by comparing the aerodynamic forces obtained from the unsteady flow simulation and predicted by the stability derivatives.

II. Theory

A. Aerodynamic Model

Without a loss of generality, we consider an aircraft with only its angle of attack α changing with time. It is well known that each of the instantaneous aerodynamic forces (the lift, for instance) is a functional of α; that is,

$$L(t) = L[\alpha(\xi)], \qquad -\infty < \xi \le t \tag{1}$$

This is to say, the instantaneous aerodynamic force depends not only on the instantaneous value of α, but also on its time history. This is one of the most important reasons that make the determination of the stability derivatives difficult. To simplify the aerodynamic model, Etkin [6] suggested expanding $\alpha(\xi)$ into a Taylor series about time t,

$$\alpha(\xi) = \alpha(t) + (\xi - t)\dot{\alpha}(t) + \tfrac{1}{2}(\xi - t)^2\ddot{\alpha}(t) + \cdots \tag{2}$$

and Eq. (1) was replaced by

$$L(t) = L(\alpha(t), \dot{\alpha}(t), \ddot{\alpha}(t), \ldots) \tag{3}$$

Equation (3) is the aerodynamic model adopted in the present paper. The $\alpha(t), \dot{\alpha}(t), \ddot{\alpha}(t), \ldots$ are used to trace the time history of $\alpha(\xi)$ before time t. It is clear that Eq. (3) is valid only if $L(t)$ satisfies

$$L(t) = L[\alpha(\xi)], \qquad \xi \in [t - \delta t, t] \subset I(t) \tag{4}$$

where $I(t)$ is the domain of convergence of the Taylor series. Therefore, if $L(t)$ depends on a sufficiently short period of the time history of $\alpha(\xi)$, and $\alpha(\xi)$ is sufficiently smooth, then Eq. (3) will be a good approximation of Eq. (1). From this analysis, the limitations of the aerodynamic model expressed by Eq. (3) are also clear. For example, this model may not be valid when the aircraft undergoes a transient aerodynamic response induced by a sudden maneuver.

The $\alpha(t), \dot{\alpha}(t), \ddot{\alpha}(t), \ldots$ are used to account for the time-history effects. In this sense, at time t, only the instantaneous values of these parameters are needed to be considered. As the instantaneous values, they can be considered to be independent of each other in Eq. (3), although they are closely related as functions of time. The independence of these parameters justifies the introduction of the stability derivatives that can be defined as

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$$\frac{\partial L(t)}{\partial \gamma} = \frac{\partial}{\partial \gamma} L(\alpha(t), \dot{\alpha}(t), \ddot{\alpha}(t), \ldots) \tag{5}$$

where γ is any one of $\alpha(t), \dot{\alpha}(t), \ddot{\alpha}(t), \ldots$.

In wind-tunnel experiments, flight test, and empirical methods, the specific form of Eq. (3) must be assumed to compute the stability derivatives. However, for some CFD-based methods using the automatic differentiation [2] or the sensitivity equations [3,4], the specific form of Eq. (3) is not needed. This is also the case for the present method.

In this paper, we consider the longitudinal stability derivatives only for simplicity. If we further assume that the magnitude of velocity is unchanged with time, the aerodynamic model of the present paper can be written as

$$L(t) = L(\alpha, \dot{\alpha}, \ddot{\alpha}, \ldots; q, \dot{q}, \ddot{q}, \ldots) \tag{6}$$

where q is the angular velocity.

B. Sensitivity Equations

We consider the two-dimensional unsteady Euler equations as the governing equations for computing the stability derivatives, which can be written as follows for the flow relative motion with respect to a frame of reference that is rigidly attached to the moving aircraft:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = R \tag{7}$$

where

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \qquad F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (\rho E + p) u \end{pmatrix}$$

$$G(U) = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (\rho E + p) v \end{pmatrix}, \qquad R = \begin{bmatrix} 0 \\ R_{Vx} \\ R_{Vy} \\ R_E \end{bmatrix}$$

and

$$R_{Vx} = -\rho[a_{ox} - 2qv - \dot{q}y - q^2 x]$$
$$R_{Vy} = -\rho[a_{oy} + 2qu + \dot{q}x - q^2 y]$$
$$R_E = -\rho u[a_{0x} - \dot{q}y - q^2 x] - \rho v[a_{0y} + \dot{q}x - q^2 y]$$

In these equations, ρ is the density, u and v are the velocity components, p is the pressure, E is the total energy, and a_{0x} and a_{oy} are the components of acceleration vector of the origin of the moving frame.

Based on the aerodynamic model (6), we can further assume that

$$U(t, x, y) = U(\alpha, \dot{\alpha}, \ddot{\alpha}, \ldots; q, \dot{q}, \ddot{q}, \ldots; x, y) \tag{8}$$

According to Eq. (8), we can distinguish two different sets of independent variables. At a fixed point (x, y) in the flowfield, if the vector of the conservative variables U is computed by solving Eq. (7), the independent variables is t; if U is computed using the aerodynamic model, the set of independent variables is $(\alpha, \dot{\alpha}, \ddot{\alpha}, \ldots; q, \dot{q}, \ddot{q}, \ldots)$. We denote the partial differential operator with respect to these two sets of independent variables with $\partial/\partial t$ and $\bar{\partial}/\bar{\partial}\gamma$, respectively, where γ is any variable belonging to the set $(\alpha, \dot{\alpha}, \ddot{\alpha}, \ldots; q, \dot{q}, \ddot{q}, \ldots)$. The relation between the partial differential operators of these two set of independent variables is

$$\frac{\partial U}{\partial t} = \frac{\bar{\partial} U}{\bar{\partial} \alpha}\dot{\alpha} + \frac{\bar{\partial} U}{\bar{\partial} \dot{\alpha}}\ddot{\alpha} + \cdots + \frac{\bar{\partial} U}{\bar{\partial} q}\dot{q} + \frac{\bar{\partial} U}{\bar{\partial} \dot{q}}\ddot{q} + \cdots \tag{9}$$

or

$$\frac{\partial U}{\partial t} = \sum_{j=0}^{\infty} \frac{\bar{\partial} U}{\bar{\partial} \alpha_j}\alpha_{j+1} + \sum_{j=0}^{\infty} \frac{\bar{\partial} U}{\bar{\partial} q_j} q_{j+1} \tag{10}$$

if we denote $(\alpha, \dot{\alpha}, \ddot{\alpha}, \ldots; q, \dot{q}, \ddot{q}, \ldots)$ with $(\alpha_0, \alpha_1, \alpha_2, \ldots; q_0, q_1, q_2, \ldots)$.

Differentiating Eq. (7) with respect to α_i and with Eq. (10) being taken into consideration, we have

$$\sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} \alpha_j}\alpha_{j+1} + \sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} q_j} q_{j+1} + \sum_j U_{\alpha_j}\frac{\bar{\partial}\alpha_{j+1}}{\bar{\partial}\alpha_i} + \sum_j U_{q_j}\frac{\bar{\partial} q_{j+1}}{\bar{\partial}\alpha_i} + \frac{\partial F_{\alpha_i}}{\partial x} + \frac{\partial G_{\alpha_i}}{\partial y} = R_{\alpha_i} \tag{11}$$

where $U_{\alpha_i} = \bar{\partial} U/\bar{\partial}\alpha_i$ and $U_{q_i} = \bar{\partial} U/\bar{\partial} q_i$. Because $(\alpha_0, \alpha_1, \alpha_2, \ldots; q_0, q_1, q_2, \ldots)$ are independent according to the previous discussions, it is apparent that $\bar{\partial}\alpha_{j+1}/\bar{\partial}\alpha_i = \delta_{i,j+1}$ and $\bar{\partial} q_{j+1}/\bar{\partial}\alpha_i = 0$, where $\delta_{i,j+1}$ is the Dirac function defined by

$$\delta_{i,j+1} = \begin{cases} 1 & \text{if } i = j+1 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Equation (11) can thus be simplified to give

$$\sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} \alpha_j}\alpha_{j+1} + \sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} q_j} q_{j+1} + \sum_j U_{\alpha_j}\delta_{i,j+1} + \frac{\partial F_{\alpha_i}}{\partial x} + \frac{\partial G_{\alpha_i}}{\partial y} = R_{\alpha_i} \tag{13}$$

We note that

$$\frac{\partial U_{\alpha_i}}{\partial t} = \sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} \alpha_j}\alpha_{j+1} + \sum_j \frac{\bar{\partial} U_{\alpha_i}}{\bar{\partial} q_j} q_{j+1} \tag{14}$$

Substituting Eq. (14) into Eq. (13) gives

$$\frac{\partial U_{\alpha_i}}{\partial t} + \sum_j U_{\alpha_j}\delta_{i,j+1} + \frac{\partial F_{\alpha_i}}{\partial x} + \frac{\partial G_{\alpha_i}}{\partial y} = R_{\alpha_i} \tag{15}$$

Similar equations with respect to q_i can be derived, which read

$$\frac{\partial U_{q_i}}{\partial t} + \sum_j U_{q_j}\delta_{i,j+1} + \frac{\partial F_{q_i}}{\partial x} + \frac{\partial G_{q_i}}{\partial y} = R_{q_i} \tag{16}$$

where U_{α_i} and U_{q_i} are the sensitivity derivatives. Equations (15) and (16) are the flow sensitivity equations with respect to α_i and q_i, respectively. When $i \geq 1$, the sensitivity derivatives are called the *dynamic sensitivity derivatives*, which are mainly affected by the unsteady effects. We note that for $i \geq 1$, there are additional terms

$$\sum_j U_{\alpha_j}\delta_{i,j+1} = U_{\alpha_{i-1}}$$

in Eq. (15) and

$$\sum_j U_{q_j}\delta_{i,j+1} = U_{q_{i-1}}$$

in Eq. (16), respectively. These terms are not presented for the $i = 0$ case. It is on this point that the present work differs from that of Limache and Cliff [3] and Limache [4], in which only the $i = 0$ case is considered.

Equations (15) and (16) can be written in more familiar forms as

$$\begin{aligned}
&\frac{\partial}{\partial t}(U_\alpha)+\frac{\partial}{\partial x}(F_\alpha)+\frac{\partial}{\partial y}(G_\alpha)=R_\alpha\\
&\frac{\partial}{\partial t}(U_{\dot\alpha})+\frac{\partial}{\partial x}(F_{\dot\alpha})+\frac{\partial}{\partial y}(G_{\dot\alpha})=R_{\dot\alpha}-U_\alpha\\
&\frac{\partial}{\partial t}(U_{\ddot\alpha})+\frac{\partial}{\partial x}(F_{\ddot\alpha})+\frac{\partial}{\partial y}(G_{\ddot\alpha})=R_{\ddot\alpha}-U_{\dot\alpha}\\
&\frac{\partial}{\partial t}(U_q)+\frac{\partial}{\partial x}(F_q)+\frac{\partial}{\partial y}(G_q)=R_q\\
&\frac{\partial}{\partial t}(U_{\dot q})+\frac{\partial}{\partial x}(F_{\dot q})+\frac{\partial}{\partial y}(G_{\dot q})=R_{\dot q}-U_q
\end{aligned}\tag{17}$$

where only the low-order sensitivity equations (up to $\ddot\alpha$ and $\dot q$) are considered.

C. Stability Derivatives

The stability derivatives can be readily evaluated after computing the sensitivity derivatives U_γ, where γ is either α_i or q_i. Taking the moment coefficient

$$C_m=\left[\oint_\Omega \boldsymbol{r}\times p\boldsymbol{n}\,\mathrm{d}s\right]\bigg/\left(\frac{1}{2}\rho V_\infty^2 SL\right)\tag{18}$$

as an example, its stability derivative with respect to γ is

$$(C_m)_\gamma=\left[\oint_\Omega \boldsymbol{r}\times p_\gamma\boldsymbol{n}\,\mathrm{d}s\right]\bigg/\left(\frac{1}{2}\rho V_\infty^2 SL\right)\tag{19}$$

where $\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}$, $\boldsymbol{n}$ is the unit outward normal vector of the aircraft surface, V_∞ is the freestream velocity, and S and L are the area and length, respectively, in the definition of the aerodynamic coefficients. In Eqs. (18) and (19), the integrations are carried out on the surface of the aircraft defined by Ω, and p_γ can be derived straightforwardly using U_γ.

So far, we have proposed a method for computing the stability derivatives of an aircraft. The new features of this method include the following:

1) The stability derivatives can be computed by the solution of the flow sensitivity equations for any maneuver (either forced maneuver or maneuver determined by the coupled solution of the aerodynamic equations and the equations of motion).

2) For any maneuver, all stability derivatives of interests can be computed without a need to computationally separate the stability derivatives from the combined derivatives.

3) The unsteady effects are taken into consideration, which makes it possible to compute any dynamic derivatives.

III. Results and Discussions

A. Test Case

Equation (7) governing the fluid flow and Eq. (17) governing the flow sensitivity derivatives are solved simultaneously by a second-order finite volume scheme. The numerical method is similar to that presented in [7] with the exception that the Harten–Lax–van Leer (HLL) [8] approximate Riemann solvers are used for both Eqs. (7) and (17). The test case is the forced oscillation of a NACA 0012 airfoil. The freestream Mach number is $M_\infty=0.5$. The airfoil oscillates about the center of mass at quarter-chord length, according to function

$$\alpha=-\theta=\alpha_0+\alpha_1\sin\left(\frac{2V_\infty kt}{c}\right)\tag{20}$$

where θ is the pitch angle ($q=\dot\theta$), α_0 is the mean angle of attack, α_1 is the amplitude of the oscillation, c is the chord length, and k is the reduced frequency. The grid number is 300×100. In the numerical simulation, we choose $\alpha_0=5$ deg and $\alpha_1=0.2$ deg. Two reduced frequencies are used, which are $k=0.0814$ and 0.0407, respectively.

We note that for a larger freestream Mach number, there are possible shock waves in the flowfield. The solution of the sensitivity equations for flow with shock waves has been studied by Appel [9]. The application of this technique in computing the stability derivatives will be the subject of our future study.

B. Stability Derivatives

The stability derivatives with respect to α, q, $\dot\alpha$, $\dot q$, and $\ddot\alpha$ will be presented in the present paper. Higher-order stability derivatives are not computed to ensure that the computational efforts are moderate so that the simulation can be run on a personal computer. We should note that the stability derivatives can be computed for any maneuver. When the maneuver is determined by Eq. (20), many theoretical and experimental methods can only evaluate the combined derivatives such as $(C_m)_{\dot\alpha}-(C_m)_q$. However, the present method can compute $(C_m)_{\dot\alpha}$ and $(C_m)_q$ separately without the need for additional efforts to separate them from $(C_m)_{\dot\alpha}-(C_m)_q$.

Figure 1 shows curves of the lift coefficient and its stability derivatives versus the angle of attack at $k=0.0814$ and 0.0407, respectively. For the present amplitude of oscillation and reduced frequency, the variations of the stability derivatives of the lift coefficient are rather small and can be effectively treated as constants. Figure 2 shows the curves of the moment coefficient and its stability derivatives versus the angle of attack at two reduced frequencies. In this case, the variations of the stability derivatives of the moment coefficient are relatively large. Therefore, some linear aerodynamic models assuming that the stability derivatives are only functions of mean angle of attack will induce larger errors. For both lift coefficient derivatives and moment coefficient derivatives, the time-lagged effects are observed in the numerical results. These effects get stronger at a higher reduced frequency.

We note that there are bumps in some curves of the stability derivatives shown in Figs. 1 and 2. These bumps are the results of the motions of the stagnation points from the windward side to the lee side of the airfoil and vice versa.

C. Verification of the Present Method

The lift and moment coefficients can be predicted by the stability derivatives. Taking the lift coefficient as an example, we have

$$\mathrm{d}C_l=\frac{\partial C_l}{\partial\alpha}\,\mathrm{d}\alpha+\frac{\partial C_l}{\partial q}\,\mathrm{d}q+\frac{\partial C_l}{\partial\dot\alpha}\,\mathrm{d}\dot\alpha+\frac{\partial C_l}{\partial\dot q}\,\mathrm{d}\dot q+\frac{\partial C_l}{\partial\ddot\alpha}\,\mathrm{d}\ddot\alpha+\cdots\tag{21}$$

The integration of Eq. (21) gives

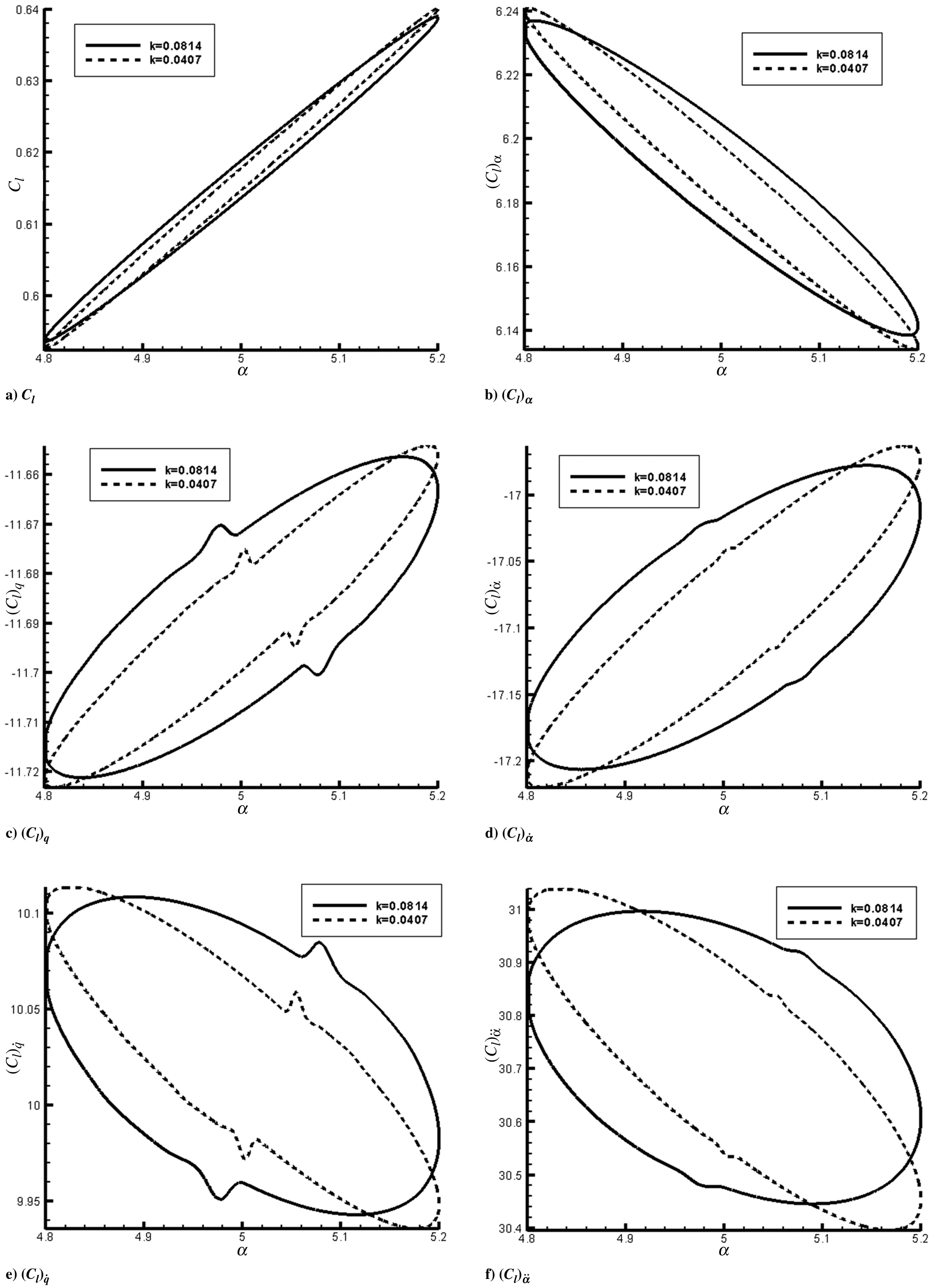
$$\begin{aligned}\Delta C_l(t)=C_l(t)-C_l(0)&=\int_{\alpha(0)}^{\alpha(t)}\frac{\partial C_l}{\partial\alpha}\,\mathrm{d}\alpha+\int_{q(0)}^{q(t)}\frac{\partial C_l}{\partial q}\,\mathrm{d}q\\
&+\int_{\dot\alpha(0)}^{\dot\alpha(t)}\frac{\partial C_l}{\partial\dot\alpha}\,\mathrm{d}\dot\alpha+\int_{\dot q(0)}^{\dot q(t)}\frac{\partial C_l}{\partial\dot q}\,\mathrm{d}\dot q+\int_{\ddot\alpha(0)}^{\ddot\alpha(t)}\frac{\partial C_l}{\partial\ddot\alpha}\,\mathrm{d}\ddot\alpha+\cdots\end{aligned}\tag{22}$$

or

$$\begin{aligned}C_l(t)\approx C_l(0)&+\int_{\alpha(0)}^{\alpha(t)}\frac{\partial C_l}{\partial\alpha}\,\mathrm{d}\alpha+\int_{q(0)}^{q(t)}\frac{\partial C_l}{\partial q}\,\mathrm{d}q\\
&+\int_{\dot\alpha(0)}^{\dot\alpha(t)}\frac{\partial C_l}{\partial\dot\alpha}\,\mathrm{d}\dot\alpha+\int_{\dot q(0)}^{\dot q(t)}\frac{\partial C_l}{\partial\dot q}\,\mathrm{d}\dot q+\int_{\ddot\alpha(0)}^{\ddot\alpha(t)}\frac{\partial C_l}{\partial\ddot\alpha}\,\mathrm{d}\ddot\alpha\end{aligned}\tag{23}$$

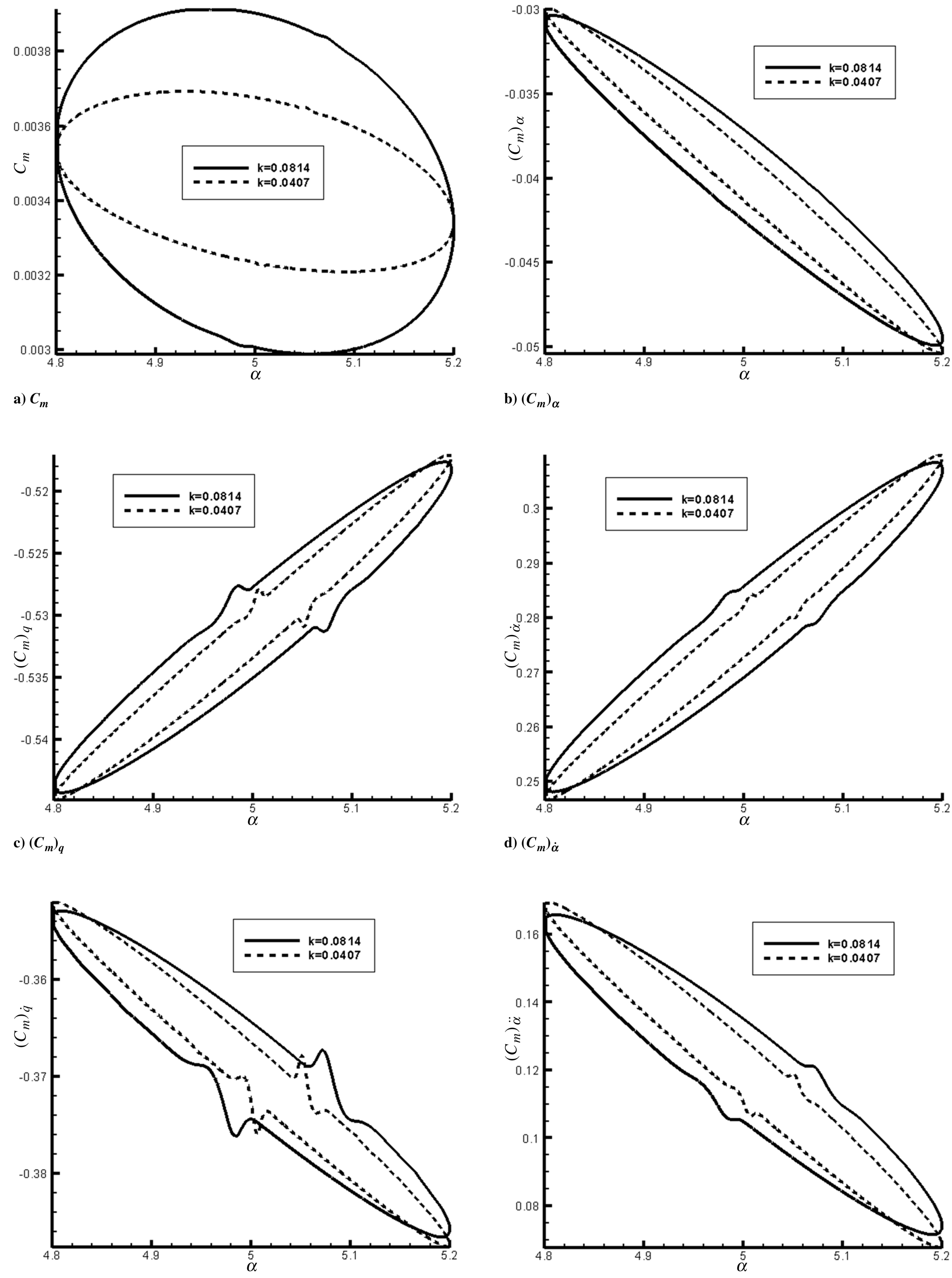
Similar results can be obtained for the moment coefficient. On the other hand, the lift and moment coefficients can certainly be predicted by unsteady flow simulation. Therefore, the consistency between these two predictions can be used to verify the present method.

Figure 3 compares the lift coefficient predicted by the flow solver with that computed by Eq. (23) for the case with $k=0.0814$. The agreement between these two predictions is fairly good. The discrepancies can be explained by the approximations made in deriving the aerodynamic model and/or the neglect of higher-order



a) C_l **b)** $(C_l)_\alpha$ **c)** $(C_l)_q$ **d)** $(C_l)_{\dot{\alpha}}$ **e)** $(C_l)_{\dot{q}}$ **f)** $(C_l)_{\ddot{\alpha}}$

Fig. 1 C_l **and its stability derivatives.**



a) C_m

b) $(C_m)_\alpha$

c) $(C_m)_q$

d) $(C_m)_{\dot{\alpha}}$

e) $(C_m)_{\dot{q}}$

f) $(C_m)_{\ddot{\alpha}}$

Fig. 2 C_m and its stability derivatives.

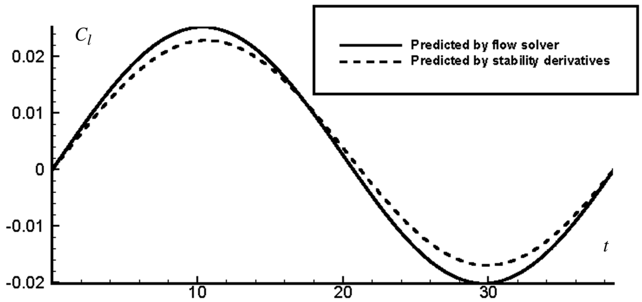


Fig. 3 Comparison of the lift coefficients predicted by the flow solver and by the stability derivatives.

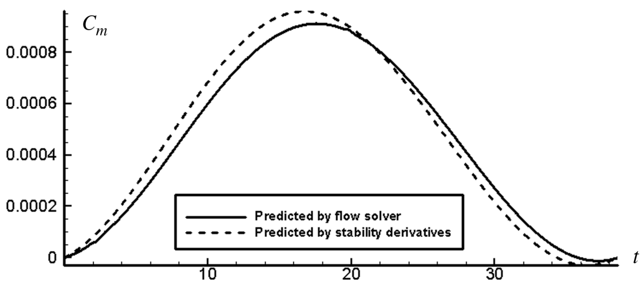


Fig. 4 Comparison of the moment coefficients predicted by the flow solver and by the stability derivatives.

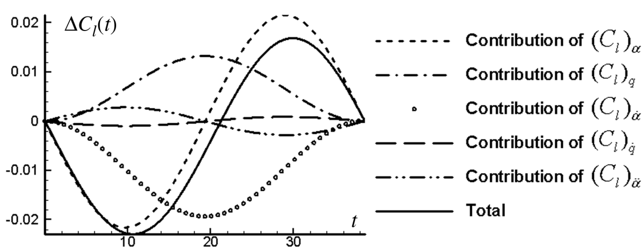


Fig. 5 Comparison of the contributions of different stability derivatives to the variation of the lift coefficient.

derivatives. The comparison between the moment coefficients predicted by the flow solver and by the stability derivatives is shown in Fig. 4 for the same test case. Good agreement can be also observed.

D. Relative Contributions of the Stability Derivatives to the Aerodynamic Forces

Equation (22) can be used to evaluate the contribution of every stability derivative to the variation of the lift coefficient, $\Delta C_l(t)$. For example, the contribution of $(C_l)_\alpha$ to the change of the lift coefficient is

$$\int_{\alpha(0)}^{\alpha(t)} \frac{\partial C_l}{\partial \alpha}\, \mathrm{d}\alpha$$

Figure 5 shows the contributions of the stability derivatives to the change of C_l when $k = 0.0814$. It can be seen that the change of C_l is mainly due to the static derivative $(C_l)_\alpha$. Although the contributions of $(C_l)_q$ and $(C_l)_{\dot\alpha}$ are also large, they are in opposite sign and tend to eliminate each other. The higher-order derivatives $(C_l)_{\dot q}$ and $(C_l)_{\ddot\alpha}$ are less important in this case.

Figure 6 shows the contributions of the stability derivatives to the variation of the moment coefficient. Unlike the lift coefficient case, the $(C_m)_\alpha$ contributes to the change of C_m less than $(C_m)_q$ and $(C_m)_{\dot\alpha}$.

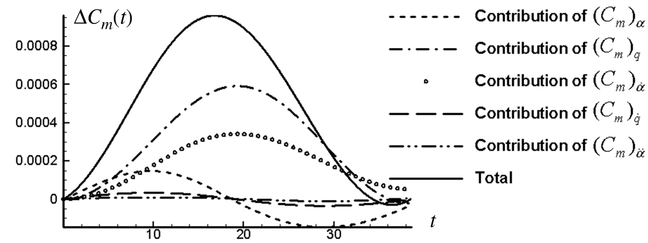


Fig. 6 Comparison of the contributions of different stability derivatives to the variation of the moment coefficient.

The dynamic derivative $(C_m)_{\dot\alpha}$ accounts for about one-third of the C_m variation. Therefore, the unsteady effect is more important for the evolution of C_m. Again, the contributions of $(C_m)_{\dot q}$ and $(C_m)_{\ddot\alpha}$ are very small.

IV. Conclusions

In this paper, a numerical method for evaluation of the stability derivatives based on the sensitivity equations is proposed. This method takes the unsteady effects into consideration and can predict all stability derivatives of interest by the numerical solutions of the unsteady flows and corresponding sensitivity equations. This method was used to compute the stability derivatives of an airfoil undergoing pitching oscillation. The agreement between the aerodynamic forces predicted by the flow solver and by the stability derivatives validates the rationality of the proposed method. The contributions of the stability derivatives to the changes of the lift and moment coefficients are analyzed. It is found that $(C_l)_\alpha$ contributes most to the change of C_l, and $(C_m)_q$ and $(C_m)_{\dot\alpha}$ are mainly responsible for the variation of C_m.

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